Math 124: Calculus I - Dr. Andy Loveless

Essential Course Info

My Course Website:	math.washington.edu/~aloveles/
Homework Log-In (use UWNetID):	webassign.net/washington/login.html
Directions for Webassign code purchase:	math.washington.edu/webassign
Math Department 124 Course Page:	math.washington.edu/~m124/

First week to do list

1. Read 10.1, 2.1, 2.2, and 2.3 of the book. Start attempting HW.
2. Print off the "worksheets" and bring them to quiz sections.

Today

- Syllabus/Intro
- Section 10.1
  - lines/circles
  - parametric motion
  - review

1st HW assignments

Closing time is always 11pm.

- 10.1 closes Oct 2 (Mon)
- 2.1 closes Oct 4 (Wed)
- 2.2, 2.3 close Oct 6 (Fri)

Expect 8-10 hrs of work,
10.1 and 2.1 take longer, start today!

What we will do in this course:

We learn the basic tools of differential calculus which provide the essential language for engineering, science and economics. Specifically,

1. **Ch. 2 – Limits and tangents,**
   Foundation of ALL calc. concepts
   - $\lim_{h\to 0} ??$, $\lim_{x\to\infty} ??$, $\frac{f(x+h)-f(x)}{h}$

2. **3.1-6, 10.1-2 – Derivative Rules**
   Key mechanical skills
   - Product, quotient, chain
   - Implicit, parametric, logarithmic
   - Notation

3. **3.9-10, Ch. 4 – Some Applications**
   - Related Rates
   - Max/Min
   - Curve Sketching

4. **Practicing Algebra, Trig and Precalc**

Students often say: The hardest part of calculus is you have to know all your precalculus, and they are right.

Improving your algebra, trig and precalculus skills will be one of the best benefits you will gain from this course (arguably as valuable as the course content itself). You will use these skills often in your other courses at UW.

Circles/Lines/Tangents and
10.1 Parametric Equation Intro

Circles: The equation describing the points (x, y) on the edge of a circle with center (x_c, y_c) and radius r is

$$(x - x_c)^2 + (y - y_c)^2 = r^2.$$

Example (you do)*:*
Give the equation of the circle centered at (2,0) of radius 5. Then answer these questions:

- Is (-1,4) on this circle?
- Is (4,3) on this circle?

$$(x-2)^2 + (y-0)^2 = 5^2$$

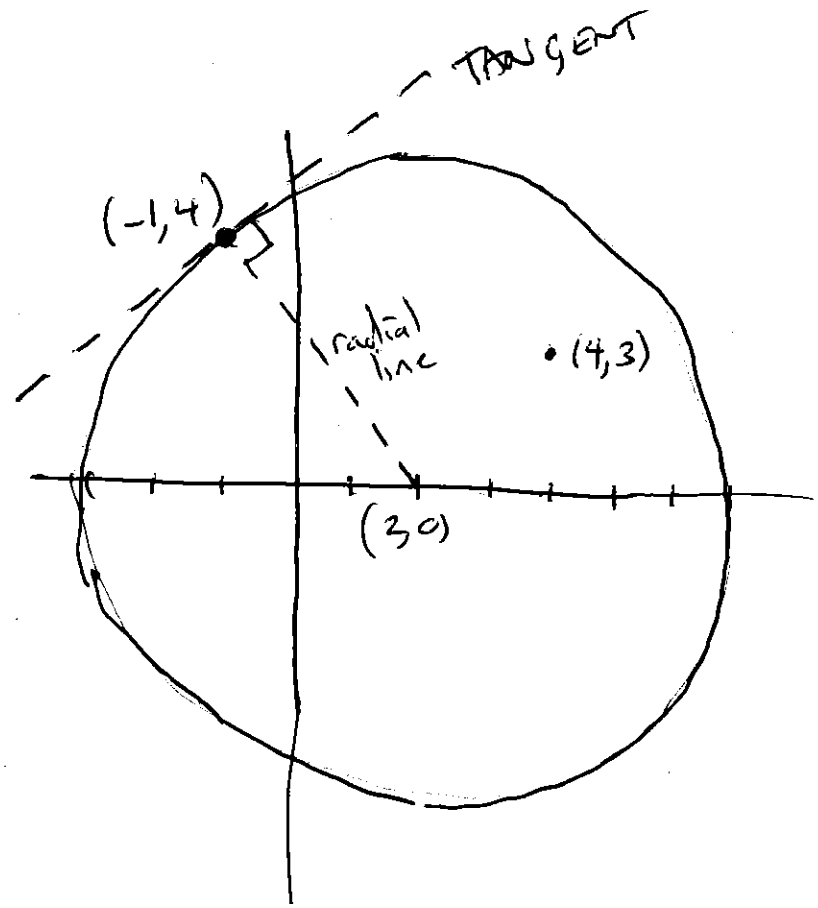
$$\boxed{(x-2)^2 + y^2 = 25}$$

IS (-1,4) ON CIRCLE?

$$(-1-2)^2 + 4^2 = 9+16 \overset{\checkmark}{=} 25 \quad \boxed{\text{YES}}$$

IS (4,3) ON CIRCLE?

$$(4-2)^2 + 3^2 = 4+9 = 13 \neq 25 \quad \boxed{\text{NO}}$$



Lines: The equation describing the points (x, y) on the line ~~with~~ through (x_0, y_0) with slope m is

$$y = m(x - x_0) + y_0$$

Example (you do):
Find the equation of the line through ~~the center of the circle~~ (-1,4) and the center of the circle from the previous example (we call this a *radial* line for the circle)

GOES THRU $(2,0)$ AND $(-1,4)$

SLOPE $= \frac{4-0}{-1-2} = -\frac{4}{3}$ $\quad \frac{y_2-y_1}{x_2-x_1}$

$\boxed{y = -\frac{4}{3}(x-2)+0}$ $\left(y \overset{\text{OR}}{=} -\frac{4}{3}(x--1)+4\right)$

SAME

Tangent Lines: A tangent line to a curve at a point is a line that "just touches" the curve at that point (a more precise definition is coming in chapter 2).

In the case of a circle, the tangent line will always be perpendicular to the radial line at that point.

Key fact: Perpendicular lines have negative reciprocal slopes.

Example (you do):

Find the equation of the tangent line to the circle of the previous examples at (-1,4).

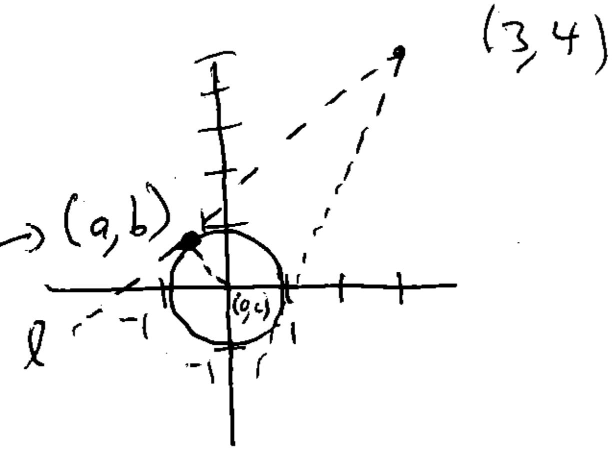
NEGATIVE RECIPROCAL

$$\text{slope} = -\frac{1}{(-4/3)} = 3/4$$

$$y = \frac{3}{4}(x - -1) + 4$$

$$\boxed{y = \frac{3}{4}(x+1) + 4}$$

An circle/line/tangent application
(just like HW)
Find the equations for all lines that are tangent to the <u>unit</u> circle and also pass through the point (3, 4).



STEP 1: LABEL UNKNOWN POINT

STEP 2: WRITE DOWN ALL THE FACTS.

(i) CIRCLE EQUATION: $x^2+y^2=1$ AND (a,b) IS ON THE CIRCLE SO

$$\boxed{a^2+b^2=1}$$

(ii) SLOPE OF RADIAL LINE: $\dfrac{b-0}{a-0} = \dfrac{b}{a}$

(iii) SLOPE OF TANGENT LINE: $-\dfrac{1}{(b/a)} = -\dfrac{a}{b}$

(iv) TANGENT LINE <u>ALSO</u> GOES THROUGH $(3,4)$ (and (a,b))

SO SLOPE $= \dfrac{b-4}{a-3}$

WANT THESE TO BE THE SAME!

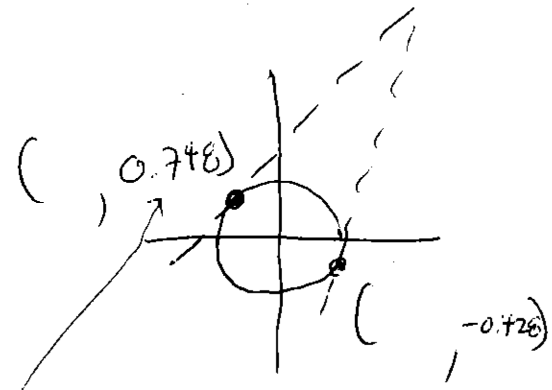
THUS, $\boxed{-\dfrac{a}{b} \stackrel{?}{=} \dfrac{b-4}{a-3}}$ $\Rightarrow -a(a-3) = b(b-4)$

$\Rightarrow -a^2+3a = b^2-4b$

$\Rightarrow 3a = a^2+b^2-4b$

STEP 3: COMBINE AND SOLVE

$$a^2+b^2=1 \Rightarrow a=\pm\sqrt{1-b^2}$$



SUBSTITUTE INTO

$$3a=a^2+b^2-4b \Rightarrow \pm 3\sqrt{1-b^2}=1-\cancel{b^2}+\cancel{b^2}-4b$$

$$\Rightarrow \pm 3\sqrt{1-b^2}=1-4b$$

$$\Rightarrow 9(1-b^2)=(1-4b)^2$$

$$\Rightarrow 9-9b^2=1-8b+16b^2$$

$$\Rightarrow 0=25b^2-8b-8$$

$$b=\frac{8\pm\sqrt{8^2-4(25)(-8)}}{2(25)}$$

$$=\frac{8\pm\sqrt{864}}{50}$$

$$b\approx 0.7478775 \Rightarrow a=-\sqrt{1-b^2}\approx -0.6638367$$

or

$$b\approx -0.4278775 \Rightarrow a\approx\sqrt{1-b^2}\approx 0.9038367$$

STEP 4: ANSWER QUESTION

$$\boxed{\begin{array}{l}(a,b)\approx(-0.6638,\ 0.7479)\\ (a,b)\approx(0.9038,\ -0.4279)\end{array}}$$

10.1 Parametric Equation Basics

We often need parametric equations when applying our calculus concepts to motion problems this term. So there are a few introductory exercises mixed into the first two homework sets.

Parametric Equations are any set of equation of the form x = x(t), y = y(t).

Basic Example 1: *Linear Motion*

$$x = x_0 + v_x t$$

$$y = y_0 + v_y t$$

$$(x_0, y_0) = \text{initial location}$$

$$v_x = \text{horizontal velocity} = \frac{\Delta x}{\Delta t}$$

$$v_y = \text{vertical velocity} = \frac{\Delta y}{\Delta t}$$

Example: The location of a bug on the *xy*-plane after *t* seconds is given by

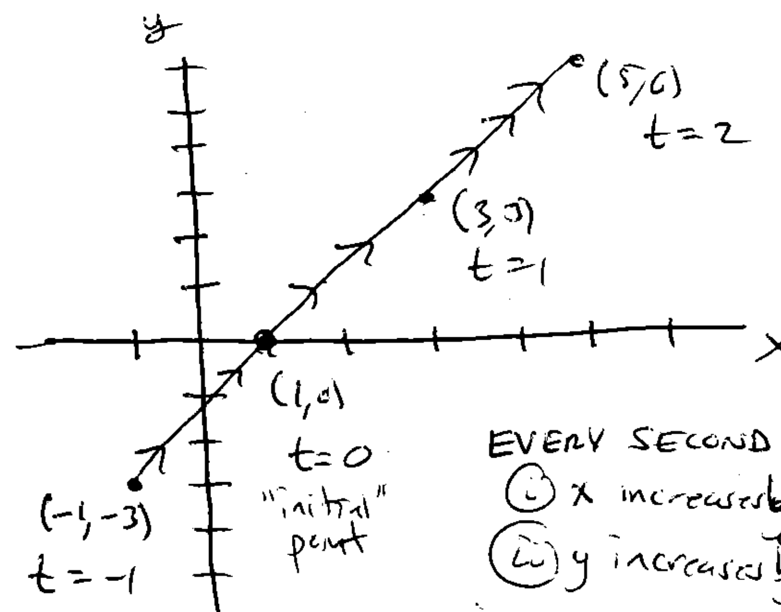
$$x = 1 + 2t \quad , \quad y = 3t$$

You do:

Plug in t = -1, t = 0, t = 1, and t = 2.

Plot these points in the xy-plane

t	-1	0	1	2
x	-1	1	3	5
y	-3	0	3	6



EVERY SECOND

(i) x increases by 2

(ii) y increases by 3

i.e. $v_x = 2$

$v_y = 3$

Basic Example 2: *Circular Motion*

$$x = x_c + r\cos(\theta_0 + \omega t)$$
$$y = y_c + r\sin(\theta_0 + \omega t)$$

$(x_c, y_c) =$ center of circle

r = radius of circle

θ_0 = initial angle

ω = angular speed $= \dfrac{\Delta\theta}{\Delta t}$

HW Note: Plot points and use these skills on problems 3, 4, 5 of 10.1 HW and 7,8 of 2.1 HW.

$\theta_0 = 3\pi/2$ ("LOWEST" PT ON CIRCLE)

$\omega = \frac{\pi}{2}$ $\frac{\text{radians}}{\text{second}}$ (90° rotation every second)

$x_c = 0$, $y_c = 3$

$r = 2$

Example: The location of an ant on the xy-plane after *t* seconds is given by

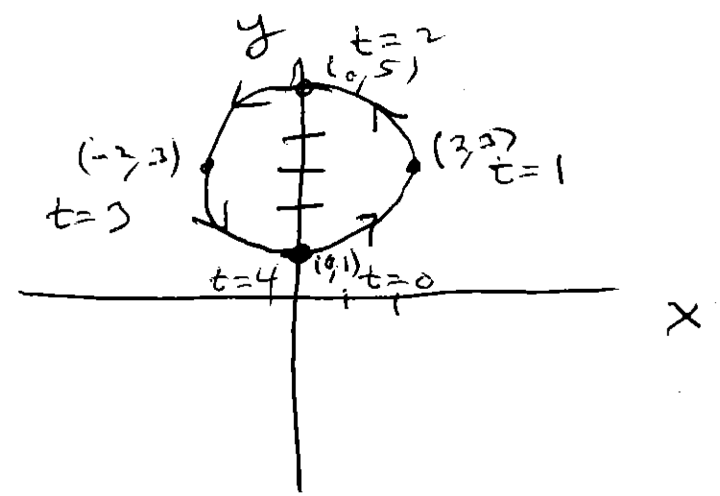
$$x = 2\cos\left(\frac{3\pi}{2} + \frac{\pi}{2}t\right)$$
$$y = 3 + 2\sin\left(\frac{3\pi}{2} + \frac{\pi}{2}t\right)$$

You do:

Plug in t = 0, t = 1, t = 2, t = 3, and t = 4.

Plot these points in the xy-plane.

t	0	1	2	3	4
x	0	2	0	-2	0
y	1	3	5	3	1
θ	$3\pi/2$	2π	$5\pi/2$	3π	$7\pi/2$



Overview of Trigonometric Functions Values and Basic Facts

If r is the radius of a circle and θ is an angle measured from standard position, then we can find the corresponding location on the edge of the circle by using the formulas

$$x = r\cos(\theta) = r\cos(\theta_0 \pm wt) \;\; and\; y = r\sin(\theta) = r\sin(\theta_0 \pm wt)$$

For most values of θ, $\sin(\theta)$ and $\cos(\theta)$ are not easily computed and require a calculator. However, you are expected to know the following values:

Angle		$\sin(\theta)$	$\cos(\theta)$
0 deg	0 rad	0	1
30 deg	$\pi/6$ rad	$1/2$	$\sqrt{3}/2$
45 deg	$\pi/4$ rad	$\sqrt{2}/2$	$\sqrt{2}/2$
60 deg	$\pi/3$ rad	$\sqrt{3}/2$	$1/2$
90 deg	$\pi/2$ rad	1	0

You can find the other trig function values at these angles using the relationships:

$\tan(\theta) = \sin(\theta)/\cos(\theta)$, $\cot(\theta) = \cos(\theta)/\sin(\theta)$, $\csc(\theta) = 1/\sin(\theta)$, $\sec(\theta) = 1/\cos(\theta)$.

Often these values are remembered by actually putting them on a circle. Here is the circle with radius 1 (or the *unit circle*) with the values at the above angles label along with corresponding angles in other quadrants. If the radius is larger, we just multiply each x and y coordinates by the radius.